

Introduction

A-Level mathematics is a very rich and rewarding course, that is extremely well valued by both employers and universities. It is, however, a challenging course. We want you to be ready to face these challenges and to do so you must be fluent in the number and algebraic skills that you studied at GCSE.

The purpose of this booklet is to inform you of the required skills for the A-Level mathematics course and the resources available to you to help you prepare.

How to Prepare

During the first couple of weeks of the course you will take a diagnostic test to assess these key number and algebraic skills. If you score less than 50%, you will be required to join additional bridging classes, in which you will continue to work on these skills alongside the new A-Level content. A sample diagnostic test is included in this booklet to help you prepare.

A useful approach to your preparation is to do the sample diagnostic test in test conditions, without the help of textbooks or additional resources. Mark your test to identify the areas that you need to work on. You can then focus your preparation on these areas, using the resources that are detailed in this booklet.

Recommended Resources

We strongly recommend purchasing and using the book by CGP "Head Start to A-Level Maths", ISBN 978-1782947922. This short book provides practice for all the key skills assessed in the diagnostic test and is available from Amazon (and other retailers) for approximately £6. You will see that the sample diagnostic test is taken from this book and the sections of the test link to the pages of the book that will support you in the corresponding areas.

Alternative resources for practice include Hegarty maths and the A-Level revision website (<https://alevelmathsrevision.com/bridging-the-gap/>). Both these excellent resources provide explanatory videos as well as plenty of practice questions. Note that the list of Hegarty tasks is extensive as it covers all aspects of the listed topics - select the tasks that are appropriate for your level.

The table in the 'Required Skills' section of this document details each of the required skills, along with details of how you can practise these skills from the recommended resources.



Types of Number and Fractions

These topics are covered in Section 1 — p.6-7.

- Which of the following are integers?
4 -3.5 0.3 $\frac{4}{5}$ 8.99 -10 205 0
- Which of the following values are rational, and which are irrational?
5.9 π $\sqrt{7}$ $\frac{1}{5}$ -6 $\sqrt{4}$ 13.978 2.1
- Evaluate the following without using a calculator, giving your answers in their lowest terms. Give any answers larger than 1 as improper fractions.
 - $\frac{2}{9} \times \frac{3}{5}$
 - $\frac{1}{6} \div \frac{2}{3}$
 - $\frac{1}{12} + \frac{5}{6}$
 - $\frac{8}{5} - \frac{1}{7}$

Indices, Multiplying Out Brackets and Factorising

These topics are covered on p.8-11.

- Simplify the following:
 - $x^7 \times x^2$
 - $10y^3 \div 5y$
 - m^0
 - $(2n^2)^5$
- Write 5^{-2} as a fraction.
- Evaluate the following without using a calculator:
 - $\left(\frac{3}{4}\right)^2$
 - $16^{\frac{1}{2}}$
 - $8^{\frac{2}{3}}$
 - $36^{-\frac{1}{2}}$
- Multiply out the brackets and simplify your answers where possible.
 - $(x + 4)(x - 6)$
 - $(x + 5)^2$
 - $(2x - 1)(x + 3)$
 - $(x + 1)(x - 4)(x + 5)$
- Factorise the following:
 - $5x + 20$
 - $3a + 12ab$
 - $x^2 - 4$
 - $9x^2 - 36$
 - $x^2 - 5$

Surds

This topic is covered on p.12-13.

- Simplify the following:
 - $\sqrt{3} \times \sqrt{2}$
 - $(\sqrt{5})^2$
 - $\frac{\sqrt{30}}{\sqrt{6}}$
 - $\sqrt{12} + 2\sqrt{3}$
 - $(1 + \sqrt{7})^2$
- Rationalise the denominators of the following:
 - $\frac{3}{\sqrt{2}}$
 - $\frac{\sqrt{5}}{2\sqrt{2}}$
 - $\frac{2}{3 + \sqrt{6}}$
 - $\frac{\sqrt{2}}{1 - \sqrt{5}}$

Solving Equations and Rearranging Formulas

You'll find these on p.14-15.

11) Solve the following:

a) $5x - 2 = 8$ b) $3(x - 6) = 2(x - 4)$ c) $\frac{x+2}{3} + \frac{2x}{5} = x+2$ d) $2x(x + 1) = 2x + 18$

12) Make x the subject of the following formulas:

a) $y = mx + c$ b) $y = \frac{3x+2}{5}$ c) $y = 2x^2z + 1$ d) $y = \frac{3x+1}{x-2}$

Quadratic Equations

Quadratics are covered
in Section 3 — p.16-21.

13) Solve the following by factorising:

a) $x^2 - 3x + 2 = 0$ b) $x^2 + 6x + 5 = 0$ c) $2x^2 - 3x - 5 = 0$ d) $3x^2 - 13x = -12$

14) Solve the following using the quadratic formula.
Give your answers to two decimal places.

a) $x^2 + 2x - 10 = 0$ b) $2x^2 - 5x - 1 = 0$

The formula is: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

15) Solve the following by completing the square. Give your answers as surds.

a) $x^2 - 4x - 2 = 0$ b) $2x^2 + 4x - 7 = 0$

16) a) Complete the square for $x^2 + 6x + 8$.

b) Hence sketch the graph of $y = x^2 + 6x + 8$, labelling the turning point and intercepts with the x -axis.

Algebraic Fractions, Inequalities and Simultaneous Equations

17) Simplify the following:

a) $\frac{15a^3b^3}{5a^2b}$ b) $\frac{2x^2y}{(4xy)^2}$ c) $\frac{x^2-16}{x^2-x-20}$

These topics are on p.22-29.

18) Simplify the following:

a) $\frac{9b^2}{a} \times \frac{2a^2}{3b}$ b) $\frac{2(x-1)^2}{15} \times \frac{10}{4x-4}$ c) $\frac{3x^2-21x}{x+2} \div \frac{x(x-7)}{9x+18}$ d) $\frac{3}{x+1} + \frac{2x-3}{x^2}$

19) Solve the following inequalities:

a) $7x + 5 \leq 2x$ b) $2(10 - x) > 4$ c) $2x^2 + 3 < 21$
d) $4x^2 - 9 \geq 7$ e) $x^2 - 4x + 10 \geq 2x + 5$

20) Draw a set of axes with the x -axis from -2 to 3 and the y -axis from 0 to 6 .
Show on these axes the region that satisfies the following inequalities:

$$y > 3x - 1, \quad y < x + 3 \quad \text{and} \quad y \geq \frac{x}{5} + 2$$

21) Solve the following simultaneous equations:

a) $2x + y = 2$ b) $3x - 2y = 1$ c) $y = x^2 + 3$ d) $3y = 2(x^2 - 3)$
 $x - 3y = 8$ $5x - 3y = 7$ $y - 2x = 18$ $2x - y = 2$



Proof and Functions

These topics are covered on p.30-33.

- 22) Prove that the sum of any three consecutive odd numbers is a multiple of 3.
- 23) Naveen says, "for any integers x and y , $xy > y$ ". Prove that Naveen is wrong.
- 24) $f(x) = \frac{x+5}{3}$ and $g(x) = x - 3$.
- a) Evaluate $f(4)$. b) Find $fg(x)$. c) Find $f^{-1}(x)$.

Straight Lines and Quadratic Graphs

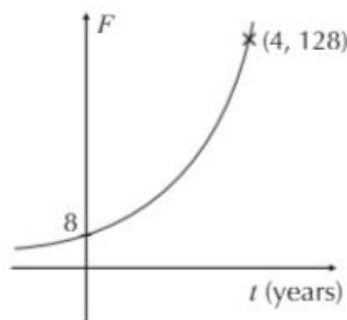
- 25) Give the gradient and y -intercept of the line $x + 2y = 4$.
- 26) Point A has coordinates (5, 2) and point B has coordinates (2, -4).
- a) Find the equation of the line passing through points A and B.
- b) Find the exact length of line AB.
- 27) Line A has equation $y = 2x + 5$.
- a) Find the equation of the line parallel to line A which passes through (3, 2).
- b) Find the equation of the line perpendicular to line A which passes through (2, 1).
- 28) Sketch the graph of $y = x^2 - 8x + 15$. Label the graph with the coordinates of the turning point and the points where the graph crosses the axes.

Go to p.34-37 if you found these questions tricky.

Harder Graphs and Graph Transformations

You can brush up on the skills needed for these questions on p.38-41.

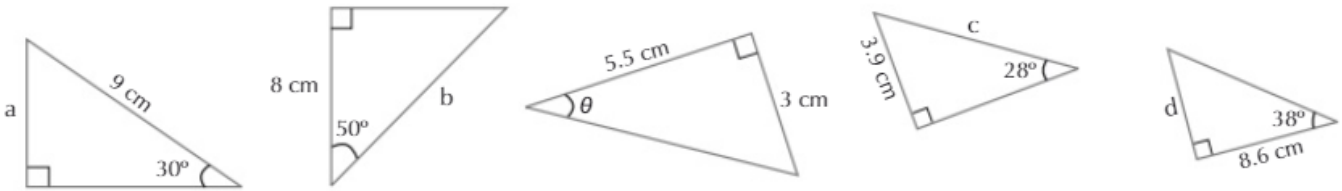
- 29) Sketch the following graphs:
- a) $y = x^3$ b) $y = \frac{1}{x}$ c) $y = -\frac{1}{x}$
- 30) The graph on the right shows how the number of fish (F) living in a river changes over time. The equation of the graph is $F = mn^t$ where t is the number of years and m and n are positive constants. Find the values of m and n .
- 31) Find the equation of the tangent to $x^2 + y^2 = 25$ at the point (3, 4).
Give your answer in the form $ax + by + c = 0$.
- 32) $f(x) = x^2$. For parts a) to c) below, sketch the graphs of $y = f(x)$ and the given transformation.
- a) $y = f(x) + 3$ b) $y = f(x + 3)$ c) $y = -f(x)$



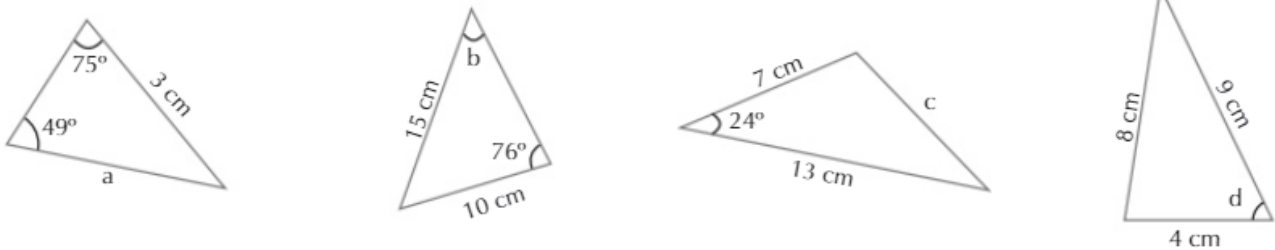
Trigonometry and Vectors

These topics are in Section 6 — p.42-50.

33) Find the unknowns in each of these triangles. Give your answers to 1 decimal place.

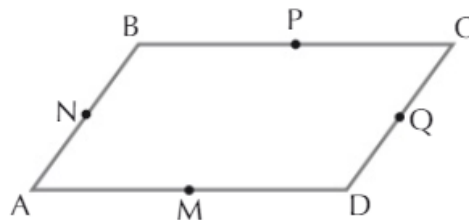


34) Find the unknowns in each of these triangles. Give your answers to 1 decimal place.

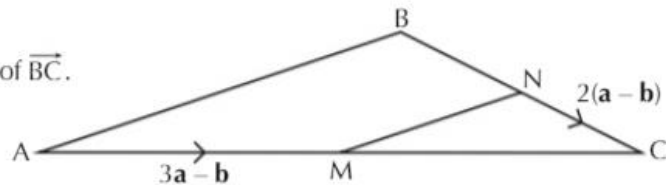


35) ABCD is the parallelogram shown on the right. M , N , P and Q are the midpoints of the sides. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$. Find the following vectors in terms of \mathbf{a} and \mathbf{b} .

- a) \overrightarrow{AC} b) \overrightarrow{DQ} c) \overrightarrow{CM}
d) \overrightarrow{QP} e) \overrightarrow{MB} f) \overrightarrow{PA}



36) The diagram shows triangle ABC. M is the midpoint of \overline{AC} and N is the midpoint of \overline{BC} . $\overrightarrow{AM} = 3\mathbf{a} - \mathbf{b}$ and $\overrightarrow{NC} = 2(\mathbf{a} - \mathbf{b})$. Show that \overline{AB} and \overline{MN} are parallel.



Sampling and Histograms

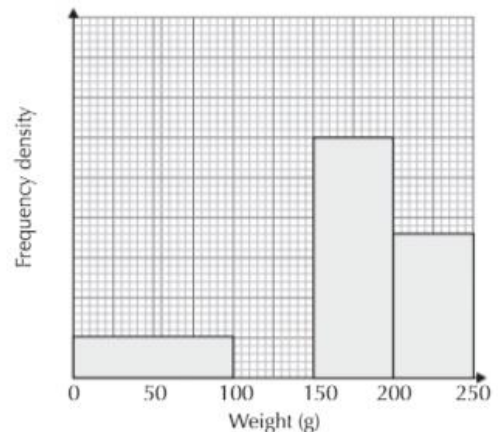
You'll find these topics on p.51-53.

37) Describe how a simple random sample of size 20 can be selected from a population of 200.

38) The weights of the chocolate bars in a shop storeroom are shown in the table and histogram below.

- a) Use the information in the table and the histogram to label the vertical axis.
b) Use the histogram to complete the table.
c) Use the table to add the missing bar to the histogram.

Weight (w , in grams)	Frequency
$0 < w \leq 100$	50
$100 < w \leq 150$	100
$150 < w \leq 200$	150
$200 < w \leq 250$	



Averages and Cumulative Frequency

Averages are covered on p.54-55.

39) Find the mean, median and mode(s) of these numbers:

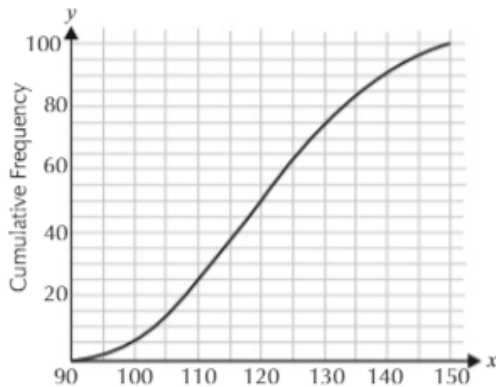
5 3 -2 0 -3 2 1 1 4 2 6 11 -4

40) The table shows the journey times between home and school for 60 students.

Time (m minutes)	Frequency
$5 < m \leq 10$	4
$10 < m \leq 15$	25
$15 < m \leq 20$	18
$20 < m \leq 25$	8
$25 < m \leq 30$	5

- a) Write down the modal class.
- b) Which group contains the median?
- c) Estimate the mean value.
- d) Draw a cumulative frequency graph for the data in the table.

41) Using this cumulative frequency graph, find the:



- a) median
- b) lower quartile
- c) upper quartile
- d) interquartile range

You can learn about cumulative frequency on p.56.

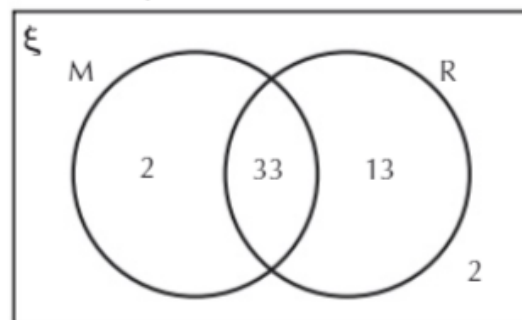
Probability and Tree Diagrams

You can learn about these topics on p.57-60.

42) Lewis asked 50 people if they like mashed potatoes (M) and roast potatoes (R). The Venn diagram shows the results.

A person is chosen at random. Find the probability that they:

- a) like mashed potatoes
- b) like neither mashed nor roast potatoes
- c) like both types of potatoes
- d) don't like roast potatoes
- e) don't like mashed potatoes



43) Mona's purse contains two £5 notes, four £10 notes and three £20 notes. It also contains five 20p coins, four 50p coins and three £1 coins.

- a) Mona picks one note and one coin at random from her purse. Find the probability that she picks a £5 note and a 20p coin.
- b) Mona picks two coins at random without replacement. Use a tree diagram to find the probability she picks a 50p coin and a £1 coin.



- 1 4, -10, 205 and 0 are integers.
2 $5.\dot{9}$, $\frac{1}{5}$, -6, $\sqrt{4}$, 13.978 and 2.1 are rational.
 π and $\sqrt{7}$ are irrational.

- 3 a) $\frac{2}{15}$
b) $\frac{1}{4}$
c) $\frac{11}{12}$
d) $\frac{51}{35}$

- 4 a) x^9
b) $2y^2$
c) 1
d) $32n^{10}$

5 $\frac{1}{25}$

- 6 a) $\frac{9}{16}$
b) 4
c) 4
d) $\frac{1}{6}$

- 7 a) $x^2 - 2x - 24$
b) $x^2 + 10x + 25$
c) $2x^2 + 5x - 3$
d) $x^3 + 2x^2 - 19x - 20$

- 8 a) $5(x + 4)$
b) $3a(1 + 4b)$
c) $(x + 2)(x - 2)$
d) $9(x + 2)(x - 2)$
e) $(x + \sqrt{5})(x - \sqrt{5})$

- 9 a) $\sqrt{6}$
b) 5
c) $\sqrt{5}$
d) $4\sqrt{3}$
e) $8 + 2\sqrt{7}$

- 10 a) $\frac{3\sqrt{2}}{2}$
b) $\frac{\sqrt{10}}{4}$
c) $\frac{6 - 2\sqrt{6}}{3}$
d) $\frac{\sqrt{2} + \sqrt{10}}{-4}$

- 11 a) $x = 2$
b) $x = 10$
c) $x = -5$
d) $x = -3$ or $x = 3$

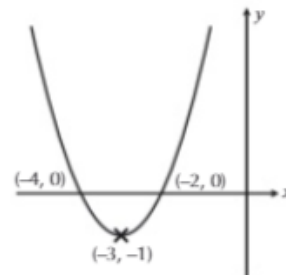
- 12 a) $x = \frac{y-c}{m}$
b) $x = \frac{5y-2}{3}$
c) $x = \pm\sqrt{\frac{y-1}{2z}}$
d) $x = \frac{2y+1}{y-3}$

- 13 a) $x = 2$ or $x = 1$
b) $x = -5$ or $x = -1$
c) $x = 2.5$ or $x = -1$
d) $x = \frac{4}{3}$ or $x = 3$

- 14 a) $x = 2.32$ or $x = -4.32$
b) $x = 2.69$ or $x = -0.19$

- 15 a) $x = 2 + \sqrt{6}$ or $x = 2 - \sqrt{6}$
b) $x = -1 + \frac{3}{\sqrt{2}}$ or $x = -1 - \frac{3}{\sqrt{2}}$

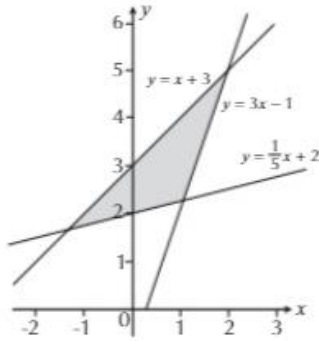
- 16 a) $x^2 + 6x + 8 = (x + 3)^2 - 1$
b)



- 17 a) $3ab^2$
b) $\frac{1}{8y}$
c) $\frac{x-4}{x-5}$

- 18 a) $6ab$
b) $\frac{x-1}{3}$
c) 27
d) $\frac{5x^2-x-3}{x^2(x+1)}$

- 19 a) $x \leq -1$
b) $x < 8$
c) $-3 < x < 3$
d) $x \leq -2$ or $x \geq 2$
e) $x \leq 1$ or $x \geq 5$



- 21 a) $x = 2, y = -2$
 b) $x = 11, y = 16$
 c) $x = -3, y = 12$ or $x = 5, y = 28$
 d) $x = 0, y = -2$ or $x = 3, y = 4$

- 22 Take three consecutive odd numbers:
 $2n + 1, 2n + 3$ and $2n + 5$, where n is an integer.
 $2n + 1 + 2n + 3 + 2n + 5 = 6n + 9 = 3(2n + 3)$
 The sum of three consecutive odd numbers can be written as $3x$, where $x = 2n + 3$.
 Therefore it is a multiple of 3.

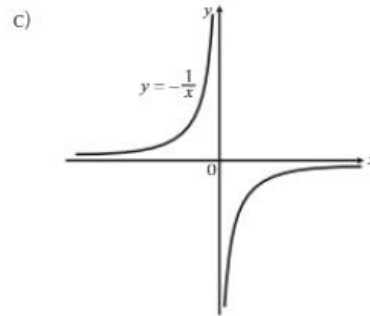
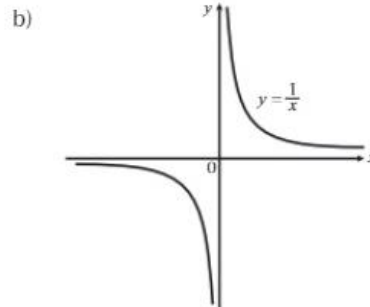
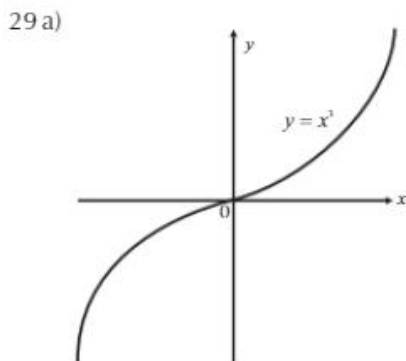
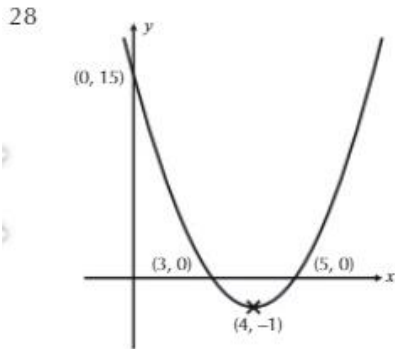
- 23 E.g. Let $x = 3$ and $y = -1$. So $xy = -3 \Rightarrow xy < y$.
 So Naveen is wrong.

- 24 a) 3
 b) $fg(x) = \frac{x+2}{3}$
 c) $f^{-1}(x) = 3x - 5$

- 25 Gradient = -0.5 , y -intercept = 2

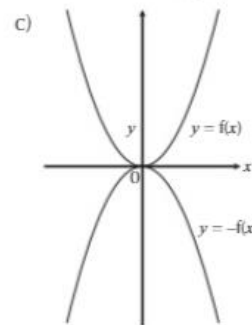
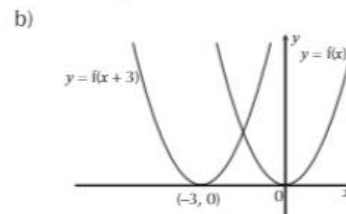
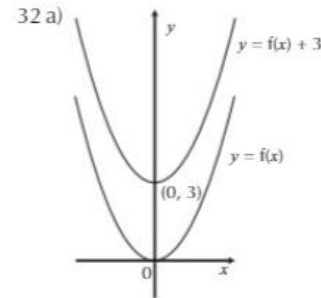
- 26 a) $y = 2x - 8$
 b) $3\sqrt{5}$

- 27 a) $y = 2x - 4$
 b) $y = -\frac{1}{2}x + 2$



30 $m = 8, n = 2$

31 $3x + 4y - 25 = 0$



33 $a = 4.5$ cm, $b = 12.4$ cm, $\theta = 28.6^\circ$,
 $c = 8.3$ cm, $d = 6.7$ cm

34 $a = 3.8$ cm, $b = 40.3^\circ$, $c = 7.2$ cm, $d = 62.7^\circ$



35 a) $\mathbf{a + b}$

b) $\frac{1}{2}\mathbf{a}$

c) $-\mathbf{a} - \frac{1}{2}\mathbf{b}$

d) $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

e) $\mathbf{a} - \frac{1}{2}\mathbf{b}$

f) $-\mathbf{a} - \frac{1}{2}\mathbf{b}$

36 $\overline{AB} = 2(3\mathbf{a} - \mathbf{b}) - 2(2(\mathbf{a} - \mathbf{b}))$
 $= 6\mathbf{a} - 2\mathbf{b} - 4\mathbf{a} + 4\mathbf{b} = 2\mathbf{a} + 2\mathbf{b} = 2(\mathbf{a} + \mathbf{b})$

$\overline{MN} = 3\mathbf{a} - \mathbf{b} - 2(\mathbf{a} - \mathbf{b}) = 3\mathbf{a} - \mathbf{b} - 2\mathbf{a} + 2\mathbf{b} = \mathbf{a} + \mathbf{b}$

$\overline{AB} = 2\overline{MN} \Rightarrow \overline{AB}$ and \overline{MN} are parallel.

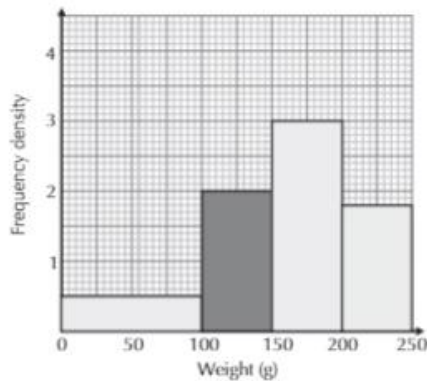
37 First assign a unique number between 1 and 200 to every member of the population. Then create a list of 20 random numbers between 1 and 200. Finally, match the random numbers to members of the population.

38 a) See histogram in part c).

b)

Weight (w , in grams)	Frequency
$0 < w \leq 100$	50
$100 < w \leq 150$	100
$150 < w \leq 200$	150
$200 < w \leq 250$	90

c)



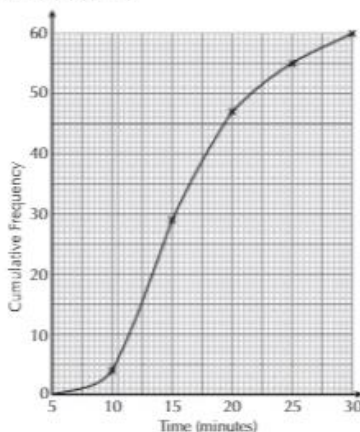
39 mean = 2, median = 2, mode = 1 and 2

40 a) $10 < m \leq 15$

b) $15 < m \leq 20$

c) 16.25 minutes

d)



41 a) 120

b) 110

c) 130

d) 20

42 a) $\frac{7}{10}$ (or 0.7)

b) $\frac{1}{25}$ (or 0.04)

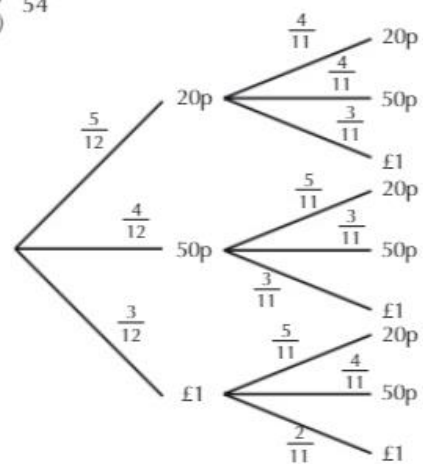
c) $\frac{33}{50}$ (or 0.66)

d) $\frac{2}{25}$ (or 0.08)

e) $\frac{3}{10}$ (or 0.3)

43 a) $\frac{5}{54}$

b)



Probability of a 50p coin and a £1 coin = $\frac{2}{11}$

Required Skills



Skill	Pages from "Head Start to A-Level Maths"	Hegarty Maths Tasks	A-Level Revision website
Number			
Types of number	6		
Fractions	7	66-71	
Basic Algebra			
Laws of indices	8-9	102-110	Topic 02 - Indices
Multiplying out brackets	10	161-166	Topic 05 - Algebra
Factorising	11	167-169, 171	
Surds	12-13	113-120	Topic 03 - Surds
Solving equations	14	178-195	
Rearranging formulae	15	280-286	Topic 01 - Rearranging Formulae
Quadratic Equations			
Factorising quadratics	16-17	223-228, 230-234	Topic 04 - Solving Quadratics by Factorisation
The quadratic formula	18	241-242	
Completing the square	19-21	235-237, 238-239	Topic 06 - Completing the Square
More Algebra			
Algebraic fractions	22	170, 172-175, 229, 244	Topic 04 - Solving Quadratics by Factorisation Topic 10 - Algebraic Fractions
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Simultaneous equations	28	246	
Proof	30	325-327	
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Graphs			
Straight lines	34	200-213	Topic 08 - Straight Lines
Parallel and perpendicular lines	36	214-219	Topic 09 - Further Straight Lines
Quadratic graphs	37	252-260	
Harder graphs	38	800-811, 299-301	
Graph transformations	40	307-313	
Trigonometry and Vectors			
Trigonometry - sin, cos, tan	42	845-853	
Trigonometry - graphs	44	302-306	
The sine and cosine rules	47	525-531	
Vectors	49	622-636	
Statistics and Probability			

Sampling	51	395	
Data basics	52	402-403	
Histograms	53	442-449	
Averages	54	404-409, 415-418	
Cumulative frequency	56	437-439	
Probability	57	351-357, 381-388	
Laws of probability	59	358-360	
Tree diagrams	60	361-367	